Read to WSU PHIL COLLOQUIUM 10/16/86

[File:Paper]
R.B.Angell
10-25-86

Transposition and Nicod's Criterion

In Hempel's "Studies in the Logic of Confirmation"1, he attacks Nicod's Criterion of Confirmation because it is incompatible with principles of transposition such as,

'If A then B' is true if and only 'if -B then -A' is true,

which Hempel considers to be logical truths, i.e., laws of logic.

Nicod's Criterion entails a concept of conditional statements which is fundamentally different from the truth-functional conditional which Hempel employs throughout. The conditional which Nicod uses is basically the same as the 'correlational conditional' or 'C-conditional' which I showed, in my last paper, could be used to express conditional probability. We saw, in that paper, that truth-functional conditionals fail utterly to express conditional probability, and that the logic of conditionals which <u>can</u> express it must not include transposition principles.

The present paper, in defending Nicod's criterion, is intended as a partial introduction to the semantics of C-conditionals, - the conditional embedded in Nicod's criterion - and show how and why, for this conditional, transposition fails to be a logical truth, as well as to explain why it is so commonly thought to express a logical truth.

I

Hempel's problem is: given a universal conditional sentence, of the form, e.g., 'For any object x: if x is P then x is Q', what would constitute confirmatory evidence? Hempel interprets the 'if...then' in the universal conditional as a truth-functional conditional, symbolized as '$(x)(Px \supset Qx)$'. But this gets him into the "paradoxes of confirmation", namely: that any observation that any object is not a Raven, (as well, we might add, as any observation of something that is black) constitutes confirmatory evidence for the generalization, "All ravens are black". This follows, in truth-functional logic, from the falsity of the antecedent: -(a is a Raven) logically implies '(a is a Raven $\supset$ a is black)', which is an instance of "All ravens are black".

Nicod's account, and the C-conditional, avoids these sorts of paradoxes. Nevertheless, Hempel rejects Nicod's account and defends the truth-functional conditional, saying that the "impression of a paradoxical situation is not objectively founded; it is a psychological illusion"2.

1See Carl G. Hempel, **Aspects of Scientific Explanation**, Free Press, 1965, pp 3-53
2Hempel, opus cit., p. 18.

II

Hempel describes Nicod's Criterion3 as follows: an object confirms the universal conditional, 'For any object x: if x is P then x is Q',

> "...if and only if it satisfies both the antecedent (here: '**P(x)**') and the consequent (here 'Q(x)') of the conditional; it disconfirms the hypothesis if and only if the satisfies the antecedent, but not the consequent; of the conditional; and (we add this to Nicod's statement) it is neutral or irrelevant, with respect to the hypothesis if it does not satisfy the antecedent..." 4

He adds,

> "We shall refer to this as Nicod's criterion. It states explicitly what is perhaps the most common tacit interpretation of the concept of confirmation."5

Hempel offers several arguments against Nicod's Criterion. Most involve truth-functional interpretations of conditionals or logical equivalence which would not obtain for C-conditionals. These can be discussed at another time. Here we discuss only the argument which, correctly, shows that Nicod's Criterion - and the logic of C-conditionals - conflicts with the widespread assumption that transposition principles like

'If A then B' is equivalent to 'if -B then -A'.
'If A then -B' implies 'if B then -A'.
'If -A then B' is true if and only if 'if -B then A' is true.
'If -A then -B' means the same as 'if B then A' .

are universal logical or semantical truths6. If we accept the C-conditional, we must deny that transposition principles are laws of logic for them.

Hempel presents this argument against Nicod7 is as follows:

> "Consider the two sentences
> S1: '(x)[Raven(x) ⊃ Black(x)]'
> S2: '(x)[-Black(x) ⊃ -Raven(x)]'
> (i.e., 'All Ravens are black' and 'Whatever is not black is not a raven'), and let **a**, **b**, c, **d** be four objects such that **a** is a raven and black, **b** a raven but not black, **c** not a raven but black, and **d** neither a raven nor black. Then according to

---

3With citation to Jean Nicod, **Foundations of Geometry and Induction** (translated by P.P.Wiener) London, 1930; p.219.
4Hempel, Opus Cit., p.11.
5Ibid.
6The right-hand formulas in these statements are called the "contrapositives" of the left-hand formulas, and vice versa.
7First presented in **Theoria**(Goteborg),v.3, (1937), p. 222.

Nicod's criterion, **a** would confirm S1, but be neutral with respect to S2; **b** would disconfirm both S1 and S2; **c** would be neutral with respect to both S1 and S2, and **d** would disconfirm S2 but be neutral with respect to S1.

"But S1 and S2 are logically equivalent; they have the same content, they are different formulations of the same hypothesis, and yet, by Nicod's criterion, either of the objects **a** and **d** would be confirming for one of the two sentences, but neutral with respect to the other."[8]

The thesis of this paper is that if you accept the concept of conditionals implicit in Nicod's Criterion, and its consequences for logic and semantics, then 'if x is a raven then x is black' is not logically equivalent to 'if x is not black then x is not a raven'; that they do not have the same content, and they are not different formulations of the same hypothesis. Hence, for that conditional, Hempel's attack fails.

We want to preserve and explicate the ordinary language expression of Hempel's "equivalence condition" for confirmation: i.e., we agree that if S3 confirms S1, and S1 is logically equivalent to S2 then S3 confirms S2. We also agree that 'S1 is logically equivalent to S2' is true if and only if 'S1 if and only if S2' is a logical truth. But the implicit concept of a conditional in Nicod's criterion, entails a different meaning for "logically equivalent" as well as for "if and only if" in these metalogical principles. What is implicit in Nicod's Criterion becomes explicit in the Logic of C-conditionals and related notions of "analytic equivalence" and "referential synonymity".

III

For purposes of this paper, I shall use (instead of 'All ravens are black') a variety of universal conditionals about what I shall call a 10L-Figures; these are figures like Figure 1 below which have 10 boxes or "locations", labeled 'L1, L2,..etc.

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
				a2	a3			a5	a6
		b5		b2	b4		b6	b3	b1

Figure 1

The 10L-Figures vary in having a's {a1,a2,a3,...}, and or b's {b1,b2,b3,...}, or in one or more, or none, of the locations. The first universal C-conditional to be considered is:

1) For any location, L, in Figure 1,
if an 'a' occurs in location L,
then a 'b' occurs in location L. [Form='If A then B']

Here we treat 'if...then' as a Nicod-, or C-conditional, and

---

[8]Hempel, **Aspects of Scientific Explanation**, p.12.

we intend that 'Figure 1' denotes the Figure 1 above, and that 'L1','L2', etc., are used to denote the rectangles in Figure 1 which contain tokens of those signs.

In this situation, though 1) clearly refers to Figure 1, what it talks about in Figure 1 are only those states of affairs in L5, L6, L9 and L10 in which there are a's; it says that in those locations there are also b's. It does not talk about anything present or absent in any other locations in Figure 1.

In contrast, consider the contrapositive of 1), the C-conditional, of the form 'If -B then -A':

1') For any location, L, in Figure 1,
if it is not the case that a 'b' occurs in location L,
then it is not the case that an 'a' occurs in location L.

What 1') talks about is not what occurs in locations picked out by the antecedent of 1) i.e., locations which have a's in them. Nor does it talk about all locations which have b's in them. It says that in locations which do not have b's, namely, L1, L2, L4, and L7, it is also the case that there are no a's.

Now it happens to be true of Figure 1 that both 1) and its contrapositive 1') are true. In other words, in this case, 1) is true of Figure 1, if and only if 1') is true of Figure 1, and that seems to be an instance of a transposition principle.

But the question is, is '1) is true if and only if 1') is true' a logical truth? They have the forms respectively, of 'If A then B' and 'If -B then -A'; are all pairs of statements having these forms logically equivalent?

IV

The term 'logically equivalent' in standard logic is equated with 1) truth-functional equivalence - wherever the truth-table for the truth-functional biconditional 'A≡B' comes out all T's, and with 2) quantificational equivalence, which is a bit more complicated, but which holds only if truth-functional equivalences hold of all of a universal quantifier's instances.

In the semantic theory I wish to advance, in place of standard truth-functional semantics, two expressions are logically equivalent if and only if they are **referentially synonymous**. We need not go into "referentially synonymous" beyond saying that if A is referentially synonymous to B, then 1) A talks about all and only the same things that B talks about, and 2) A says all and only the same things about whatever it talks about that B says about those things and vice versa. Clearly, not all truth-functionally equivalent statements are referentially synonymous; notably the pairs {'(Fa.-Fa)', '(Gb.-Gb)'} and {'(A.-A.B)', '(A.-A.-B)'}.

In the semantics of C-conditionals, truth-functional equivalents are not always "logically equivalent" in this sense. For 'A is logically equivalent to B' is true iff and only the statement 'A if and only if B' is logically true; but when 'if and only if' is taken as a C-biconditional rather than a truth-functional biconditional, "logically equivalent" gets a different meaning[9].

Thus for 1) and its contrapositive, 1'), to have the same meaning, in the sense of being referentially synonymous, they must talk about and refer to the same entities. This is what they do not do. The statement 1) talks about only those states of affairs in L5, L6, L9 and L10 which contains a's in Figure 1, while the statement 1') talks about only the states of affairs in locations which do not have b's, namely, L1, L2, L4, and L7. Hence 1) and 1') do not talk about the same things, hence do not mean the same thing (in the sense of referentially synonymy), and hence do not "have the same content" and are not "different formulations of the same hypothesis" (to use Hempel's words). Since they are not referentially synonymous, on this theory of logic they are not logically equivalent.

V

The point can also be made syntactically in terms of the calculus of analytic equivalence - i.e., in terms of its rules for theoremhood. Within standard logic it is possible to distinguish syntactically a sub-class of truth-functionally equivalent pairs, which I shall call the class of "analytically equivalent" pairs. Let the calculus AEQ, have '&' and '-' as primitive sentence connectives, the usual definitions and rules of formation, the axiom schemata,

AEQ1.	A aeq (A&A)	[&-Idempotence]
AEQ2.	(A&B) aeq (B&A)	[&-Commutation]
AEQ3.	(A&(B&C)) aeq ((A&B)&C)	[&-Association]
AEQ4.	(A&(BvC)) aeq ((A&B)v(A&C))	[&v-Distribution]
AEQ5.	--A aeq A	[Double Negation]

and the rule of transformation,

R1. If A aeq B, then C aeq C(B//A)[10]

In this system, two expressions can not be analytically equivalent if either contains a variable the other does not, or if there is an atomic sentence which occurs negatively, or positively, in one but not in the other[11].

9
On the other hand, all logically true truth-functional biconditionals remain logical truths since they are always negations of inconsistent statements; this is not the same as to say the two components are logically equivalent in our new sense.

10 For 'A aeq B' read "statement of the form A is analytically equivalent to statement of the form B"; for 'C(B//A)' read "a statement like C except that one or more occurrences of B in C are replaced by A".

11 A component occurs negatively, if and only if, in primitive notation it occurs within the scope of an odd number of negation signs; otherwise it occurs positively. Due to Herbrand.

By this criterion of "analytic equivalence" 1) is not analytically equivalent to 1'), because the atomic components of 1) and 1') occur positively in 1) but not in 1') and negatively in 1') but not in 1).

All analytically equivalent pairs in standard logic are can be proved to be referentially synonymous and thus logically equivalent in our sense. They are also all truth-functionally equivalent; but, as mentioned, not all truth-functionally equivalent pairs are analytically equivalent, and thus not all such pairs are logically equivalent in our sense. Hence, there is a formal proof that 1) and 1') are not logically equivalent in the logic of C-conditionals.

VI

It might thought be that although 1) and 1') are <u>counter-examples</u> (using C-conditionals), for the statements,

'If A then B' is <u>referentially synonymous</u> to 'If -B then -A'
'If A then B' is <u>analytically equivalent</u> to 'If -B then -A'

(thus the contrapositives are not always logically equivalent), that perhaps we could never have 'If A then B' <u>true</u> and 'If -B then -A' <u>not true</u> in the same context. Let us express this principle as

'If A then B' is <u>true</u> <u>if and only if</u> 'If -B then -A' is <u>true</u>.

In the case of Figure 1, we can see that though 1) and 1') are not logically equivalent by the definitions involving referential synonymy and analytic equivalence, nevertheless, they were both true together; one was true if and only if the other was true in the case of Figure 1. The question now is, could it <u>ever</u> be the case that one was true and the other was not true? In other words, may it not be that semantically at least, they are truth-functionally equivalent?

'Figure 1', as used in this paper, is the name of an actual state of affairs on a piece of paper in the actual world. Different conditional statements can be made with reference to Figure 1, some of which would be true, and some of which would be false. The <u>meaning</u> of a C-conditional does not depend on its being true or false. The following is a false C-conditional:

2) For any location, L, in Figure 1,
<u>if</u> an 'b' occurs in location L,
<u>then</u> a 'a' occurs in location L.

for (following Nicod's Criterion), this picks out the locations L3, L5, L6, L8, L9, and L10, which have b's in them, and says that they all have a's in them, which is false. The <u>meaning</u> of the statement 2) would have been the same, however, had Figure 1 been drawn differently so as to make 2) true. The fact that Figure 1 is just what it is, is what makes 2) false..

Figure 1

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
				a2	a3			a5	a6
		b5		b2	b4		b6	b3	b1

But it also seems clear with respect to Figure 1, that if 2) is false, then its contrapositive,

2') For any location, L, in Figure 1,
<u>if</u> it is not the case that an 'a' occurs in location L,
<u>then</u> it is not the case that a 'b' occurs in location L.

must be false also. Thus in Figure 1 it seems that when a statement is false, its contrapositive must be false also. In other words,

'If A then B' is false <u>if and only if</u> 'If -B then -A' is false.

apparently holds for Figure 1; and this also seems to reinforce this new formulation of transposition.

If the conditional is <u>true</u>, then <u>there exists</u>, in some context being referred to, a set of two or more locations, at least one of which contains the antecedent, and all locations which contain the antecedent contain the consequent. The question now is, Whenever this is the case, must the contrapositive conditional also be true?

But a problem arises. In the semantics of C-conditionals, if no location in the context referred to contains the antecedent, then the conditional can not be true, for then it is talking about something that isn't there, in the field of reference[12]. Nor can it be false; for to be false in the semantics of the C-conditional is to satisfy the antecedent, but not the consequent. A C-conditional can be neither true nor false in some cases.

Now consider Figures 2 and 3 and the statements 3) and 3') below. The statement 3) has the form 'If A then B' and is true of Figure 2 below but is not true of Figure 3; and the statement 3') has the form 'If -B then -A' and is true of Figure 3, but is not true of Figure 2. This situation is due in each case to the fact that the antecedent is not satisfied: what is being talked about does not exist in the field of reference.

---

[12]This principle, that a C-conditional is neither true nor false when referred to contexts in which the antecedent is not satisfied, is an extension of the property it must have when serving as the conditional of conditional probability; the probability of B given A, is undefined when the probability of A is 0. This semantic property also comes in handy, however, in dealing with other problems like problems relating to non-referring terms, or inconsistent predicates. It presupposes a semantics which distinguish 'is false' from 'is not true' however; a plausible semantics which is demanded by C-conditionals.

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
				a2	a3			a5	a6
b8	b7	b5	b10	b2	b4	b9	b6	b3	b1

Figure 2

3) For any location, L, in Figure 2,
if an 'a' occurs in location L,
then a 'b' occurs in location L. [Form:'If A then B']

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
				b2	b4			b3	b1

Figure 3

3') For any location, L, in Figure 3,
if it is not the case that a 'b' occurs in location L,
then it is not the case that an 'a' occurs in location L.
[Form:'If -B then -A']

The statement 3) has the same form and the same meaning as 1) (where meaning does not include contingent actual facts about the contents of 'Figure 1' or 'Figure 2' or 'Figure 3'). But 3), and thus 1), can not be said by virtue of their meanings to imply the truth or falsehood of 3') or of 1'), for though both contrapositives are true of Figure 3, they are not true (or false) of Figure 2, since the antecedent does not apply to anything in the field of reference.

Again, consider the contrapositives, 4) and 4'); 4') is true of Figure 3, but 4) is not true, since the antecedent is not satisfied:

4) For any location, L, in Figure 4, [Form:'If A then -B']
if an 'a' occurs in location L,
then it is not the case that a 'b' occurs in location L.

4') For any location, L, in Figure 4, [Form:'If B then -A']
if a 'b' occurs in location L,
then it is not the case that an 'a' occurs in location L.

Or, consider the contrapositives, 5) and 5'); 5) is true in Figure 2, but 5') is not true of Figure 2 since its antecedent is not satisfied:

5) For any location, L, in Figure 4,
if it is not the case that an 'a' occurs in location L,
then a 'b' occurs in location L.

5') For any location, L, in Figure 4,
if it is not the case that a 'b' occurs in location L,
then an 'a' occurs in location L.

Again, because their antecedents can not be satisfied, neither one of 4) and 4') can be true in Figure 4, though they are each contrapositives of the other, having the forms 'if A then -B' and 'if B then -A' respectively:

Figure 4

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10

And likewise, neither one of 5) or 5'), which are contrapositives of each other, with the forms, 'If -A then B' and 'If -B then A', can be true of Figure 5, since neither antecedent is satisfied.

Figure 5

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
a1	a2	a3	a4	a5	a6	a6	a7	a9	a10
b1	b2	b3	b4	b5	b6	b7	b8	b9	b10

From these considerations it is clear that even where 'If A then B' is true, 'if -B then-A' may not be true, and thus that transposition for C-conditionals fails even if only truth-functional equivalence is being claimed.

Thus we have shown that none of the following forms of transposition principles are laws in the logic of C-conditionals:

A) 'If A then B' is referentially synonymous with
'(if -B then -A)'
B) 'If A then B' is analytically equivalent to
'(if -B then -A)'3
C) 'If A then B' is true if and only if
'(if -B then -A)' is true.
D) 'If A then B' is truth-functionally equivalent to
'If -B then -A'

Of course if 'if...then' is interpreted throughout as the truth-functional conditional, then all of A) through D) are true. For, the conditionals in quotes are then taken in both standard logic and analytic logic to mean '(-AvB)' and '(Av-B). Thus A) and B), for example, become, analytically equivalent to

A') '(-A v B)' is referentially synonymous with
'(--B v -A)'

B') '(-A v B)' is analytically equivalent to
'(--B v -A)'

and these are easily proven as theorems in analytic logic, or in the logic of referential synonymy. If the 'if and only if' in C) is treated as a truth-functional biconditional as well as the 'if...then's in quotes in C), then C) would be derivable in the truth-functional semantics of standard logic. Further, with truth-functional interpretaions of the 'if...then's in D), D) is a metatheorem of standard logic - it says the same thing as what is meant by standard logicians when they say ''If A then B' is logically equivalent to '(if -B then -A)''.

Despite these results, it can be shown, that **Modus Tollens** is is provable in a logic of C-conditionals with a truth-operator. I.e., "If it is true that (If A then B) and it is true that -B, then it is true that -A" can be sustained. Since the logic of the C-conditional also avoids Exportation, the failure of transposition principles will not lead to a rejection of **Modus Tollens**[13]. The proofs of this point will be not be give here.

VII

The principles of transposition have a very strong intuitive appeal. Most people, I think, would say that if 'If A then B' were true, then 'If -B then -A' would have to be true also. On the other hand, the account of conditionals given above is also very close to what people would say was meant by a conditional statement - including the notion that a conditional is neither true nor false where its antecedent does not apply to anything. In my opinion, this conditional is closer to a common sense account of the conditionals than the truth-functional account which makes every conditional true if its antecedent is false, or if its consequent is true. How can these two incompatible intuitions be explained?

We may begin by noting that in all possible Figures in which the the set of locations includes both some locations in which the antecedent is true and others in which it is not true, and similarly for the consequent, both C) and D) will be true even if the 'if and only if' in C) is taken as a C-biconditional. Among the possible 10L-Type Figures with a's and b's distributed among the locations, by far the greater number of possibilities will be those in which some locations have a's and some don't and some locations have b's and some don't[14].

In general, when people assert universal conditionals both the antecedent and the consequent, taken singly, describe states of affairs which hold in some locations (at some times, in some contexts) and do not hold in others. Even in standard quantified logic, the range of the individual variables is generally taken to refer to the set of all entities whatever, and it is often asserted that no significant predicates[15] (or, in any case, extremely few) either apply, or fail to apply, to all entities.

---

[13] I.e., 'If (T(If A then B)) & T(-B)) then T-A)'
does not imply, in the logic of C-conditionals,
'(If T(If A then B) then (If T(-B) then T-A))'
The elimination of Exportation is also necessary to get a conditional which expresses conditional probability.

[14]
Of the 1,048,576 possible 10L-Type Figures, 4092 would have all, or no a,s or b's, making one or both of the contrapositives not true, or not-false, or not-true when the other was true, or not-false when the other was false. This is less than 4/10s of 1% of the total possibilities.

[15] 'significant predicates' exclude tautologous or inconsistenct predicates.

Thus it may be suggested that the reason why the layman has difficulty thinking of counter-examples to principles of transposition is because it would rarely be the case that he would think of an example in which either the antecedent or the consequent held, or failed to hold, universally in his field of reference.

A second reason perhaps, is that many people in contemporary semantics insist upon equating 'is false' with 'is not true', and reject in the name of 'excluded middle' the possibility of meaningful indicative sentences which are both not true and not false. Movements like "Free logic", and others, are treated at the present time as 'deviant', and have not yet gained universal acceptance due to the immense power and effectiveness of standard mathematical logic and its generally accepted semantic theories.

VIII

The problem which I would like to present to logicians is whether they want 1) a logic of conditionals which entails a variety of "paradoxes" (of which the "paradoxes of confirmation" is but one sub-class) and preserves transposition as a law of logic, or 2) a logic of conditionals which eliminates this and many other "paradoxes, but does not include transposition and exportation among its laws. Or, putting the question more generously: would it not be worthwhile to recognize, in addition to the "truth-functional conditional" another kind of conditional, the C-conditional, with its own logic and a different semantics which allows us to avoid many, if not all, of the "paradoxes" of the the truth-functional conditional, at the small price of giving up some principles currently considered laws of logic?

Miscellaneous left-overs:

We will discuss this by investigating the truth or falsity of the following statements:

1) 'If A then B' truth-functionally equivalent to
   'If -B then -A'
2) 'If A then B' is analytically equivalent to
   '(if -B then -A)'
3) 'If A then B' is referentially synonymous with
   '(if -B then -A)'
4) 'If A then B' is true if and only if
   '(if -B then -A)' is true.
5) ''If A then B'is true' is analytically equivalent to
   ''(if -B then -A)' is true'.
6) ''If A then B'is true' is referentially synonymous with
   ''(if -B then -A)' is true'.

5) and 6) have not been discussed: they depend on a semantics of the truth-operator, and its relation to SYN and AEQ.

For, 'If A then B' is talking about A's and claiming a correlation between As and Bs. But 'If -B then -A' is talking about very different things non-Bs, and trying to correlate them with non-As. This brings on the Raven Paradox.

To verify or confirm (x)(If x is R then x is B) we confine our attention to Ravens; this is what the claim is about, what the sentence as a whole refer to - ravens and their properties. To talk about non-black things is to not to talk about ravens.